Grade 5 Mathematics, Quarter 1, Unit 1.1

Comparing and Rounding Whole Numbers and Decimals to the Thousandths

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Recognize place value in terms of ten times and 1/10 of the adjacent numbers.
- Explain patterns found when multiplying by powers of 10.
- Use whole number exponents to write powers of 10.
- Read and write decimals to the thousandths using base ten numerals, number names, and expanded form.
- Comparing decimals to thousandths using <, =, and > symbols.
- Round decimals to any place.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Recognize the concept of 10 times and 1/10 of adjacent numbers without calculating.

Attend to precision.

- Accurately calculate, compare, label, and explain—using their own reasoning—the place value of numbers and whole number decimal combinations.
- Use mathematical symbols consistently and appropriately when comparing two decimals.

Look for and express regularity in repeated reasoning.

- Look for regularity and trends to make sense of digit values and the place value structure.
- Identify patterns established when multiplying and dividing numbers by tens.

Essential questions

- What is the effect on place value when a given number is multiplied or divided by ten?
- How are place value patterns repeated in large numbers?
- How can whole number exponents be written to show powers of ten?
- How do you read and write whole numbers and decimals to the thousandths in standard form, expanded form, and in words?
- How do you compare any two decimal numbers to the thousandths using the symbols <, >, or =?
- How do you round a decimal number to any place value?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten 5.NBT

Understand the place value system.

5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

5.NBT.3 Read, write, and compare decimals to thousandths.

a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.

b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

5.NBT.4 Use place value understanding to round decimals to any place.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grade 3, students used place value to understand rounding of whole numbers to the nearest 10 or 100. In grade 4, students recognized whole number place value and could represent it up to 1,000,000. They understood <, >, = and the rounding of whole numbers to 1,000,000. Additionally in grade 4, students extended their understanding of fractions equivalence and comparisons, built fractions from unit fractions, and understood the decimal notation for fractions.

Current Learning

In grade 5, students recognize, write, and compare whole numbers using >, <, and =; they extend the number system to include decimals to thousandths. Students recognize patterns in the placement of whole numbers and decimals when they are multiplied or divided by powers of 10. Students use place value understanding to round decimals to any place.

Future Learning

In grade 6, students will extend previous understanding to include integers. Students will recognize rational numbers on a number line and will understand how operations can affect the directions traveled on the number line. They will understand ordering and absolute value of rational numbers.

Additional Findings

According to *A Research Companion to Principles and Standards for School Mathematics*, (p. 90) "Deep comprehension of place value and of the regular ten-for-one trades to the left as numbers get larger can help students understand decimal fractions as regular one-for-ten trades to the right as quantities get smaller."

According to PARCC, (p. 24) "The extension of the place value system from whole numbers to decimals is a major intellectual accomplishment involving understanding and skill with base ten units and fractions."

Grade 5 Mathematics, Quarter 1, Unit 1.2

Measurement and Conversions; Multidigit Multiplication and Division

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Convert units in the same measurement system and use them in multistep problems.
- Multiply multidigit whole numbers.
- Illustrate, explain, and connect calculations of multidigit whole numbers, with decimals to hundredths, in equations, rectangular arrays, and/or area models.
- Use strategies based on place value, properties of operations, and the relationship of multiplication and division to divide whole numbers to get whole number quotients.
- Understand the relationship between multiplication and division to solve problems.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Use several strategies to understand and check the answers of multiplication and division problems.
- Understand the relationship between multiplication and division.

Attend to precision.

- Specifically label units of measure within the context of the problem.
- Explain why the unit is appropriate.

Look for and make use of structure.

- Identify patterns established when multiplying and dividing numbers by tens.

Essential questions

- How would you convert a measurement into an equal and appropriate unit measure in a real-world problem?
- When solving conversion problems, how do decide which operation to use?
- When solving real-world problems, how do you determine if a measurement conversion is needed?
- How would you demonstrate and explain the process of multiplying multidigit whole numbers using the standard algorithm?
- How you would explain the relationship between multiplication and division?
- How does your understanding of place value help you solve division problems?
- What strategies help with solving a division problem?
- How would you illustrate and explain calculations by using equations, rectangular arrays, and/or area models.

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data 5.MD

Convert like measurement units within a given measurement system.

5.MD.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Number and Operations in Base Ten 5.NBT

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.5 ~~Fluently~~ multiply multi-digit whole numbers using the standard algorithm.

5.NBT.6 Find whole-number quotients of whole numbers with up to ~~four-digit dividends and two-digit divisors~~, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In grade 3, students used place value to understand rounding of whole numbers to the nearest 10 or 100. In grade 4, students used the measurement system and recorded equivalent measures within same system i.e., in a two-column table show 12 in. = 1 ft. Students understood and multiplied four digits by one-digit and two by two-digit. They divided by four digit dividend and one-digit divisors, including problems in which remainders were interpreted. Students were also introduced to letters representing an unknown quantity.

Current Learning

In grade 5, students convert and use standard measurement units. These conversions are used to solve multistep real-world problems. They multiply multidigit whole numbers using the standard algorithm. Other methods can be taught (e.g., arrays, decomposition by place value) working up to the standard algorithm. The students understand division using whole numbers and can show the relationship between multiplication and division. They use different strategies to understand division.

Future Learning

In grade 6, students will apply knowledge of measurement systems to ratios and geometry. Students will apply knowledge of multiplication and division into algebraic expressions. They will extend their understanding into multiplication and division of fractions by fractions.

Additional Findings

According to *Curriculum Focal Points,* (p. 34) "Students solve problems that require attention to both approximation and precision of measurement."

According to PARCC, (p. 24) "Students work with volume as a measurement quality to relate volume to multiplication and division."

According to *Principles and Standards for School Mathematics,* (p. 151) "Students should focus on the meaning of and relationship between multiplication and division."

Students apply their understanding of models for divisions, place value, properties, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multidigit dividends. They select appropriate methods and apply them accurately to estimate quotients or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including the standard algorithm, for dividing whole numbers, understand why the procedures work (on the basis of place value and properties or operations), and use them to solve problems (*Curriculum Focal Points*, p. 17).

Grade 5 Mathematics, Quarter 1, Unit 1.3

Making Sense of Cubic Units and Understanding Volume of Solid Figures

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Understand volume as an attribute of a solid figure.
- Understand the concept of unit cubes.
- Measure volume with unit cubes in whole units.
- Measure volume with unit cubes filling a solid without gaps or overlaps.
- Use appropriate units to measure volume. (i.e., cubic centimeters, cubic inches, improvised units).

Mathematical practices to be integrated

Model with mathematics.

- Students use proportional reasoning to apply the correct unit of measure.
- Students extend their knowledge of addition to add layers to find volume.

Use appropriate tools strategically.

- Students can identify and use the proper tools (i.e., rulers, nets, cubes), to find volume.

Essential questions

- What is volume?
- What are the attributes of a unit cube?
- What are unit cubes used to measure?
- How do you use the "unit cube" to measure volume?
- How can you use addition or multiplication and unit cubes to find volume of a solid figure?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data 5.MD

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

Common Core Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In grade 3, students were introduced to the concept of liquid volume using standard units such as liters. In grade 4, students used the measurements of liquid volumes to solve word problems.

Current Learning

In grade 5, students recognize volume attributes. They understand that volume can be measured by finding the total number of same size units of volume required to fill the space without gaps or overlaps. Students use appropriate cubic units of measure when labeling volume.

Future Learning

In grade 6, students will apply the formulas for volume to rectangular prisms with fractional edge lengths.

Additional Findings

According to PARCC, (p. 24) "Students work with volume as an attribute of a solid figure and as a measurement quantity."

According to *Principles and Standards for School Mathematics,* (p. 172) "Students should become familiar with the common units in these number systems and establish mental images or benchmarks for judging and comparing size."

Students recognize volume as an attribute of three-dimensional space. They understand that they can quantify volume by finding the total number of same-sized units of volume that they need to fill the space without gaps or overlaps. (*Curriculum Focal Points*, NCTM, p. 17)

Grade 5 Mathematics, Quarter 2, Unit 2.1

Generate and Analyze Number Patterns

Overview

Number of instructional days: 5 (1 day = 90 minutes)

Content to be learned

- Generate two numerical patterns using two given rules.
- Identify apparent relationships between terms.
- Form ordered pairs from two given rules.
- Graph ordered pairs on a coordinate plane.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Attend to the meaning of quantities, beyond simply computing them.
- Use symbolic representations and manipulate the symbols.

Construct viable arguments and critique the reasoning of others.

- Make conjectures and justify conclusions.
- Construct arguments using concrete references.
- Critique work for validity.

Model with mathematics.

- Write expressions to describe situations.
- Apply proportional reasoning to analyze a problem.
- Routinely interpret results in the context of the situation.

Essential questions

- How can you generate numerical patterns using two given rules?
- What is the relationship between the two patterns?
- How would you graph the patterns?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking 5.OA

Analyze patterns and determining and graphing relationships

5.OA.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Common Core State Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition

equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In third grade, students identified and explained patterns using properties of operations. In fourth grade, students generated number or shape patterns that followed a given rule, and they identified apparent features of the pattern not explicit in the rule.

Current Learning

At a developmental level, fifth-grade students generate two numerical patterns using two given rules. They make a T-Chart using the numbers generated by the two rules, and they also graph on a coordinate plane. Relationships between corresponding terms are formed into ordered pairs and graphed on a coordinate plane.

Future Learning

In sixth grade, students will take their knowledge of patterns and apply it to ratios and proportional reasoning.

Additional Findings

According to *A Research Companion to Principles and Standards for School Mathematics*, "systematic experience with patterns can build up to an understanding of the idea of function, and experience with numbers and their properties lays a foundation for later work with symbols and algebraic experiences." (p. 37)

According to *PARCC Framework,* "students build on their previous work with number lines to use two perpendicular number lines to define the coordinate system." (p. 24)

Grade 5 Mathematics, Quarter 2, Unit 2.2

Build and Evaluate Numerical Expressions; Multidigit Multiplication and Division

Overview

Number of instructional days: 10 (1 day = 90 minutes)

Content to be learned

- Evaluate (not solve) expressions with one of the following symbols: parentheses, brackets, or braces.
- Interpret numerical expressions then write simple expressions.
- Write simple expressions.
- Fluently multiply multi-digit numbers using the standard algorithm.
- Divide four-digit dividends by two-digit divisors.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Analyze givens, constraints, relationships, and goals.
- Explain the meaning of a problem and look for entry points to its solution.
- Monitor and evaluate progress and change course if necessary.

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships in problem situations.
- Attend to the meaning of quantities, not just how to compute them.

Essential questions

- Why would you use grouping symbols to evaluate an expression?
- How would you use grouping symbols to evaluate an expression?
- How do you multiply multi-digit numbers?
- How do you divide a four-digit dividend by a two-digit divisor?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking 5.OA

Write and interpret numerical expressions.

5.OA.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

5.OA.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7).*

Number and Operations in Base Ten 5.NBT

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.

5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Common Core State Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—

and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Clarifying the Standards

Prior Learning

In Grade 4, students used strategies based on place value and the properties of operations. They made verbal statements about multiplication equations as comparisons. Students multiplied a whole number of up to four digits by a one-digit whole number and multiplied two, two-digit numbers. Students found a whole number quotient and remainder with up to four-digit dividends and a one-digit divisor.

Current Learning

In Grade 5, at the developmental level, students write simple expressions using parentheses, brackets, or braces to interpret and evaluate mathematical expressions. At the reinforcement and practice levels, students fluently multiply multi-digit whole numbers. Students find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors.

Future Learning

In Grade 6, students will apply the properties of operations to generate equivalent expressions. Students will divide multi-digit numbers and fluently multiply multi-digit decimals. Students fluently divide multi-digit numbers using standard algorithms.

Additional Findings

According to the *Common Core Progressions*, "the generality of the concepts involved in operations and algebraic thinking means that students' work in this area should be designed to help them extend arithmetic beyond whole numbers and understand and apply expressions and equations in later grades." (OA, p. 2)

According to *Principles and Standards for School Mathematics*, "using area models, properties of operations such as commutativity of multiplication become more apparent. Research also suggests that by solving problems that require calculations, students develop a method of computing and also learn more about operations and properties." (p. 152)

According to *Curriculum Focal Points*, "students use the meaning of fractions, multiplication, and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explain why they work." (p.17)

Grade 5 Mathematics, Quarter 2, Unit 2.3

Adding and Subtracting Fractions with Unlike Denominators

Overview

Number of instructional days: 10 (1 day = 90 minutes)

Content to be learned

- Add and subtract fractions with unlike denominators.
- Solve word problems involving addition and subtraction of fractions with unlike denominators.
- Make a line plot using given data involving fractions.
- Solve problems using information presented in line plots.
- Use benchmark fractions and numbers to estimate the reasonableness of your answers.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Construct viable arguments to justify the reasonableness of answers and argue the reasonableness of others' answers.

Model with mathematics.

- Apply knowledge of operations with fractions to solve real-world problems.
- Use information from line plots to solve problems.

Attend to precision.

- Carefully select units of measure and plot data accurately.
- Label axes appropriately to convey meaning.

Essential questions

- How do you find the sum of fractions with unlike denominators?
- How do you find the difference of two fractions with unlike denominators?
- How do you solve problems involving information presented in line plots?
- How can you use benchmark fractions to estimate your answers?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions 5.NF

Use equivalent fractions as a strategy to add and subtract fractions.

5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Measurement and Data 5.MD

Represent and interpret data.

5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In fourth grade, students added and subtracted fractions with like denominators and solved word problems involving fractions with like denominators. Students solved problems based on information presented in a line plot using fractions and addition and subtraction.

Current Learning

In Grade 5, students add and subtract fractions with unlike denominators (including mixed numbers), using equivalent fractions to produce like denominators. Students solve word problems involving fractions with unlike denominators. Students use addition and subtraction of fractions to construct line plots using fractional units of measure. This is a critical content area.

Future Learning

In Grade 6, students will multiply and divide fractions by fractions. Students will build and solve real-world problems involving fractions divided by fractions. Students will extend fractional understanding to ratios and conversion of measurement units.

Additional Findings

According to the *Common Core Progressions*, in grades 3-5, work with data is closely related to the number line, fraction concepts, fraction arithmetic, and solving problems that involve the four operations.

According to *Principals and Standards for School Mathematics*, in the elementary grades, when students are talking about data, students might say that one group has more or less of some attribute than another.

Grade 5 Mathematics, Quarter 2, Unit 2.4

Understanding Fractions and Making Sense of Multiplication of Fractions

Overview

Number of instructional days: 10 (1 day = 90 minutes)

Content to be learned

- Interpret a fraction as division of the numerator by the denominator.
- Solve word problems involving division of whole numbers resulting in fractional products.
- Compare the size of a product to the size of a factor, based on the size of the other factor.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and relationships in problem situations.
- Attend to the meanings of quantities, not just how to compute them.

Construct viable arguments and critique the reasoning of others.

- Understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- Make conjectures and build a logical progression of statements to explore the truth of their conjectures.

Attend to precision.

- Use clear definitions in discussion with others and in their own reasoning.
- Give carefully formulated explanations to each other.

Essential questions

- Why is a fraction the same as division?
- Why does multiplying a given number by a fraction greater than 1 result in a product greater than the given number?
- How do you solve word problems involving division of whole numbers leading to fractional answers?
- How, if using fractions, would you compare the size of a product to the size of one factor on the basis of the size of the other factor?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions 5.NF

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

5.NF.5 Interpret multiplication as scaling (resizing), by:

a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers.

Common Core State Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.

Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In fourth grade, students understood fractions as parts of a whole and multiplied a fraction by a whole number.

Current Learning

In fifth grade, at the developmental level, students interpret a fraction as division of a numerator by the denominator. Students solve word problems involving whole numbers that result in fractions. Students interpret multiplication as scaling (resizing).

Future Learning

In Grade 6, students will interpret and compute quotients of fractions and solve division word problems involving division of fractions by fractions.

Additional Findings

According to *PARCC Framework*, "students learn the relationship between fractions and division, allowing them to divide any whole number by any nonzero whole number and express their answer in the form of a fraction or mixed number" (p. 23) and "students understand that multiplying a proper fraction can result in an answer less than the number it is multiplied by as well as understand that a multiplying an improper fraction can result in an answer greater than the number it is multiplied by." (p. 24)

Grade 5 Mathematics, Quarter 3, Unit 3.1

Multiplication of Fractions and Problem Solving

Overview

Number of instructional days: 15 (1 day = 90 minutes)

Content to be learned

- Interpret the product as a part of a partition.
- Generate and explain the product of $(a/b) \times q$ as equivalent to $a \times q \div b$.
- Find the area of rectangle with fractional side lengths.
- Solve real-world problems using the multiplication of fractions and whole numbers.
- Use fraction models and/or a story to show the products are equivalent.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and their relationships.
- Attend to meaning of quantities, beyond simply computing them.

Construct viable arguments and critique the reasoning of others.

- Construct arguments using objects, drawings, diagrams, and actions.

Attend to precision.

- State meanings of symbols they choose including using the equal sign consistently and appropriately.

Essential questions

- How can you prove that $(a/b) \times q$ is equivalent to $a \times q \div b$?
- How do you find the area of a rectangle with fractional side lengths using a visual model or equations?
- How do you find the product of a fraction and mixed numbers given in a word problem?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions **5.NF**

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Common Core State Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In fourth grade, students only multiplied fractions by whole numbers, used visual fraction models, and solved real-world problems. In third and fourth grades, students found the area of rectangles whole number by whole number. In fourth grade they are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Current Learning

At a developmental level, fifth grade students are learning to multiply fractions by fractions using visual area models. They are also reinforcing their understanding of multiplication of whole number times a fraction. They are also developing an understanding of area models with fractional sides.

Future Learning

In sixth grade, at a developmental level, students will learn the concept of dividing fractions by fractions with the use of visual models. At developmental level, they will learn to divide fractions by fractions and apply it to real-world problems and use area models to illustrate results.

Additional Findings

According to *Principles and Standards for School Mathematics*, "By using area models of fractions, by which part of a region is shaded, students can see how fractions are related to a unit whole, compare fractional parts of a whole, and find equivalent fractions" (p. 150).

According to PARCC Framework, "Students have, since grade 1, been using language such as "third of" to describe one part when a whole is partitioned into three parts. With their new understanding of the connection between fractions and division, students now see that 5/3 is one third of 5, which leads to the meaning of multiplication by a unit fraction: 1/3 x 5 = 5/3" (p. 11).

Grade 5 Mathematics, Quarter 3, Unit 3.2

Understanding Division of Fractions

Overview

Number of instructional days: 10 (1 day = 90 minutes)

Content to be learned

- Use visual fraction models and story context to explain division of unit fractions by whole numbers.
- Use visual fraction models and story context to explain division of whole numbers by unit fractions.
- Use the relationship between multiplication and division to explain that $a \div (b/c) = d$ because $2 \times (1/5) = 4$.
- Create a story problem by using $a \div (b/c)$.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*

Look for and make use of structure.

- See complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.

Look for and express regularity in repeated reasoning.

- Use the meaning of fractions of multiplication and division and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense.

Essential questions

- How could you explain the division of fraction by a non-zero number?
- How would you use a visual fraction model to show the quotient in a fraction being divided by a non-zero number?
- How would you create a story context for a whole number divided by a fraction?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions **5.NF**

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.7 Apply and extend previous understandings of dvision to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]

[1] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*

Common Core State Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

At a developmental level, fourth grade students multiplied whole numbers by a fraction using denominators 2, 3, 4, 6, 8, 10, 12, and 100.

Current Learning

At a developmental level, fifth grade students interpret division of a unit fraction by a non-zero whole number, and compute. They also interpret division of a whole number by a unit fraction. A unit fraction is a fraction whose numerator is always 1.

Future Learning

At a developmental level, sixth grade students will divide a fraction by a fraction.

Additional Findings

According to *A Research Companion to Principles and Standards for School Mathematics,* "Sharing (or partitioning) is the action of distributing an amount of something among a number of recipients so that each recipient receives the same amount." (p. 108)

According to Progressions, "Students have, since grade 1, been using language such as 'third of' to describe one part when a whole is partitioned into three parts. With their new understanding of the connection between fractions and division, students now see that 5/3 is one third of 5, which leads to the meaning of multiplication by a unit fraction." (p. 11)

Grade 5 Mathematics, Quarter 3, Unit 3.3

Problem Solving with Division of Fractions

Overview

Number of instructional days: 10 (1 day = 90 minutes)

Content to be learned

- Solve real-world problems involving division of unit fractions by non-zero whole numbers.
- Solve real-world problems involving division of whole numbers by unit fractions.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Use a visual model to solve division of whole numbers by unit fractions.
- Use a visual model to solve division of unit fractions by whole numbers.

Attend to precision.

- Use an equation to solve division of whole numbers by a fraction model.
- Explain their solutions by using appropriate academic vocabulary (i.e., unit fractions and whole numbers).

Look for and make use of structure

- Take a division equation and contextualize it to make sense of what is being asked.

Essential questions

- What is a unit fraction?
- How is a fraction different from a whole number?
- When would you need to divide a whole number by a unit fraction in real-life?
- When would you need to divide a unit fraction by a whole number in real-life?
- How can you divide a whole number by a unit fraction?
- How can you divide a unit fraction by a whole number?
- How can a visual model help you solve a division problem involving unit fractions?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations—Fractions 5.NF

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]

[1] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Common Core State Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In grade 4, students solved multiplication problems involving fractional groups of whole numbers. Students referred to unit fractions as landmark fractions in previous grades.

Current Learning

Students in grade 5 are reinforcing the meaning of fractions, multiplication and division to solve division problems involving unit fractions by using visual fraction models and equations Unit fractions are fractions in which the numerator is always 1.

Future Learning

In grade 6, students will reinforce division of fractions by extending the number system to include negative integers and rational numbers. Students will use the meaning of fractions to explain why procedures for the division of fractions make sense.

Additional Findings

In the *PARCC Progressions 3–5 Number and Operations—Fractions*, the authors state that students should solve real-world story problems using visual models to make meaning of division of fractions" (p. 12).

Principles and Standards for School Mathematics states, "Students can develop a deep understanding of rational numbers through experiences with a variety of models such as fraction strips." Studentsneed concrete representations of abstract ideas (p. 217).

Grade 5 Mathematics, Quarter 3, Unit 3.4

Perform and Explain Operations of Decimals to the Hundredths

Overview

Number of instructional days: 15 (1 day = 90 minutes)

Content to be learned

- None listed.

Mathematical practices to be integrated

- None listed.

Essential questions

- None listed.

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten **5.NBT**

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Common Core State Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In grade 4**,** students performed operations with decimals up to hundredths. They recognized that comparisons of decimal numbers are only valid when referring to the same whole.

Current Learning

In grade 5, students will compute sums, differences, products and quotients of decimals to the hundredths place efficiently and accurately.

Future Learning

In grade 6, students will extend fluency within the four operations, using standard algorithms, involving numbers with decimals.

Additional Findings

In the *PARCC Progressions K–5 Number and Operations in Base Ten*, the author states that understanding computations in the base-ten system will support later learning (p. 19).

In *Principles and Standards for School Mathematics*, the authors believe that students who can understand the structure of numbers can use them flexibly (p. 150).

Grade 5 Mathematics, Quarter 4, Unit 4.1

Developing the Formula of Volume and Relating Volume to Multiplication and Addition with Solid Figures

Overview

Number of instructional days: 12 (1 day = 45–60 minutes)

Content to be learned

- Find the volume of a right rectangular prism with whole numbers by packing it with cubes.
- Show that the volume is the same as would be found by multiplying the edge lengths, and by multiplying the heights by the area of the base.
- Represent threefold whole-number products' volumes to represent the associative property of multiplication.
- Apply the formulas $V = l \times w$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of real-world mathematical problems.
- Recognize volume as additive.
- Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding volumes of the non-overlapping parts; apply this technique to solve real-world problems.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Decontextualize by abstracting a given situation and representing it symbolically.
- Create coherent representation of the problem at hand.

Use appropriate tools strategically.

- Detect possible errors by strategically using estimation to find volume.
- Consider available tools when finding volume of right rectangular prism.

Look for and make use of structure.

- Sort a collection of shapes based on the amounts of sides or shapes they have.
- See complicated things as a single object or as being composed of several objects.

Essential questions

- How do you find the volume of a rectangular prism using cubes?
- How do you use the area of the base to find the volume of a rectangular prism?
- How do you apply the formulas $V = l \times w \times h$ and $V = B \times h$ to rectangular prisms with whole number edge lengths?
- How do you find the volume of a solid figure composed of two right rectangular prisms?

Written Curriculum

Common Core State Standards for Mathematical Content

Measurement and Data	**5.MD**

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using

estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In grade 4**,** students used the four operations to solve word problems involving liquid volume.

Current Learning

In grade 5, students find the volume of a right rectangular prism with whole number side lengths.

Future Learning

In grade 6, students will find the volume of a right rectangular prism with fractional edge lengths.

Additional Findings

In the *PARCC Progressions K–5 Geometry*, the authors state that significant challenges to students' spatial structuring include the complexity of the nature of the materials measured and that volume introduces a third dimension. That is, solid units are "packed," such as cubes in a three-dimensional array, whereas a liquid "fills" three-dimensional space, taking the shape of the container (p. 26).

According to *Curriculum Focal Points*, students' understanding that a cube is one unit on an edge is the standard unit for measuring volume (p. 17).

Grade 5 Mathematics, Quarter 4, Unit 4.2

Plotting Ordered Pairs on the Coordinate Plane

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Locate coordinates on the coordinate plane using ordered pairs.
- Using real-world problems, graph points on the first quadrant of the coordinate plane.
- Interpret results of the graph.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Explain graph data and search for regularity or trends.
- Make conjectures about the form and meaning of the solution.

Model with mathematics.

- Make assumptions and approximations to simplify a complicated situation.
- Map relationships using tools such as diagrams, two-way tables, graphs, flow charts, and formulas.

Attend to precision.

- Label axes to clarify the correspondence with quantities in a problem.
- Use clear definitions in discussion with others and in their own reasoning.

Essential questions

- How do you plot ordered pairs on a coordinate plane?
- How do you plot points given in real-world problems on a coordinate plane?
- How do you identify ordered pairs on a coordinate plane?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry 5.G

Graph points on the coordinate plane to solve real-world and mathematical problems.

5.G.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).

5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such

tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards

Prior Learning

In grade 4**,** students drew points, lines segments, rays, angles, and perpendicular and parallel lines.

Current Learning

In grade 5, students use perpendicular number lines to define a coordinate system with the intersection of the lines (the origin) arrange to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinate.

Future Learning

In grade 6, students will draw polygons in a coordinate plane, given coordinates for the vertices.

Additional Findings

In the *PARCC Progressions K–5 Geometry*, the authors state that students' competencies in shape composition and decomposition, and especially in spatial structuring of rectangular arrays, need to be highly developed because they form a foundation for understanding multiplication, area, volume, and the coordinate plane (p. 16).

According to *Curriculum Focal Points*, students begin to apply their understanding of whole numbers, fractions, and decimals as they construct and analyze double-bar and line graphs and use ordered pairs on a coordinate grids (p. 17).

Grade 5 Mathematics, Quarter 4, Unit 4.3

Classifying/Defining 2-D Figures

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Identify the attributes of two-dimensional figures.
- Classify the hierarchy of two-dimensional figures based on their properties.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Know and flexibly use different properties of objects to identify and classify two-dimensional shapes.
- Create a coherent representation of two-dimensional shapes.

Construct viable arguments and critique the arguments of others.

- Understand and explain two-dimensional shapes based upon stated assumptions and definitions.

Look for and express regularity in repeated reasoning.

- Notice if attributes are repeated and look for both general methods and shortcuts in identifying and classifying two-dimensional shapes.

Essential questions

- What attributes do two-dimensional figures have?
- What is the hierarchy of two-dimensional figures based on their properties?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry 5.G

Classify two-dimensional figures into categories based on their properties.

5.G.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*

5.G.4 Classify two-dimensional figures in a hierarchy based on properties.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention

to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grade 4**,** students classified two-dimensional figures based on the presence or absence of parallel or perpendicular lines.

Current Learning

In grade 5, students classify two-dimensional polygons by their attributes and classify them according to their knowledge.

Future Learning

In grade 6, students will use prior knowledge of two-dimensional polygons to identify polygons built on a coordinate plane.

Additional Findings

According to *Principles and Standards for School Mathematics*, students in fifth grade should develop more precise ways to describe shapes, focusing on identifying and describing the shape's properties and learning specialized vocabulary associated with these shapes and properties (p. 165).

In *Curriculum Focal Points*, the authors state that students must relate two-dimensional shapes to three-dimensional shapes by describing their numbers of edges, faces, or vertices, and analyze properties of polygons (p. 17).